APPENDIX. NOTATION

The following letter symbols conform essentially with American Standard Letter Symbols for Hydraulics (ASA—Z10.2—1942) and with ASCE *Manual of Engineering Practice No. 22* on "Soil Mechanics Nomenclature":

B = "hydraulic resistance" of formation, head loss per unit discharge;
b = thickness of confined sand bed;
C = coefficient in term $C\,Q^2$ expressing "well loss," a component of drawdown, the other term of which is $B\,Q$;
k = transmission constant, or "coefficient of permeability";
n = porosity of sand;
Q = discharge of well;
ΔQ_i = increment of discharge; $i = 1, 2, 3, \cdots$;
r = radial distance from axis of well;
r_w = "effective radius" of well;
S = "coefficient of storage" = $(b/V)(dV_w/ds)$;
s = drawdown at distance r, the difference between initial head and head at time t at that distance;
s_w = drawdown at r_w, according to theoretical logarithmic distribution;
$\Delta s^{(i)}$ = increment of drawdown produced by ΔQ_i;
$\Delta s^{(i)}/\Delta Q_i$ = "specific incremental drawdown" during i^{th} period of test;
$\Delta s^{\circ}/\Delta Q_0$ = limiting value of specific incremental drawdown for discharge approaching zero (= B);
T = "transmissibility" of sand bed = $k\,b$;
t = time;
t^* = "inflectional time" = $r^2\,S/4\,T$;
u = $r^2\,S/4\,T\,t = t^*/t$, a nondimensional variable;
V = volume;
V_w = volume of water;
$W(u)$ = "well function" of u, or the negative "exponential integral" of $-u$, for which tables are available;
α = "compressibility" of solid skeleton of sand bed, relative decrease in thickness per unit increase of vertical component of compressive stress in sand bed;
β = compressibility of water in sand bed;
β' = apparent compressibility of water = $\beta + \alpha/n$; and
γ = specific weight of water.

DISCUSSION

N. S. BOULTON,[11] ESQ.—The importance of carefully recording both the small variations in pumping level, which may occur during pumping tests at constant discharge, and the duration of the test, are appropriately stressed in this paper. From such information it is possible to predict, as the author has shown, the probable steady decline in specific capacity "for periods of several months or a few years" when the well is pumped at constant discharge. It is important to remember, however, that the accuracy of this prediction depends essentially on the assumption that the compressibility of the aquifer (which enters into the coefficient of storage) has the same value for the very small pressure releases which occur at large distances from the pumped well as for the comparatively large pressure releases near to the well. It would be appreciated if the author could present evidence in support of this assumption, based on long-period observations of declining well levels. In addition, it would be interesting to know whether the author has been able to check the values for "well loss" by direct estimates of the pipe friction loss as the water flows inside the well casing and also of the loss of head due to the screen.

For the fourth period of the test at Bethpage, Long Island, N. Y., the depth of water in the well was apparently about 238 ft. Allowing for the water entering the well uniformly along the bottom 50 ft, a reasonable estimate (from a usual formula) for the head lost in pipe friction in the 8-in.-diameter tube is about 10.5 ft, including 1.5 ft for the velocity head. The computed well loss (see heading, "Data from Multiple-Step Drawdown Test") is stated to be 15.5 ft, which leaves 5 ft for the loss due to the screen. It is easy to calculate the latter loss on the assumption of flow through a uniform permeable medium outside the screen to which Darcy's law may be applied. Thus, for long vertical slots spaced equally around the circumference of the well, it can be shown from the potential solution for the flow net that the head loss due to the restricted inlet area provided by a slotted tube is closely given by:

$$h = \frac{Q}{2\,\pi\,N\,b\,k}\log_e\left(\frac{2}{1-\cos\nu\,\pi}\right) \qquad (26)$$

in which N is the number of vertical slots around the circumference of the tube; and ν is the slot-width ratio or width of slot divided by the distance between the centers of two adjacent slots.

According to Eq. 26, the head loss is proportional to the discharge and, for a given slot-width ratio, inversely proportional to the number of slots.

If $Q = 3.39$ cu ft per sec and $b = 50$ ft, as in the fourth period of the Bethpage test, and if $k = 0.004$ ft per sec (as deduced from Fig. 8), assuming $\nu = \frac{1}{4}$ (since the dimensions of the slotted tubing are not given in the paper), it is found on substitution in Eq. 26 that $h = 5.2/N$ ft. For one hundred slots, each 0.063 in. wide, uniformly spaced around the circumference, $h = 0.052$ ft which is negligible. On the other hand, if the slots are arranged in batteries

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numbering, say, ten in the circumference, the batteries being 0.5 in. wide with 2 in. between them, $h = 0.52$ ft which is still small.

It should be emphasized that this calculation makes no allowance for any clogging of the slots. Such clogging may account for the discrepancy between the small calculated screen loss and the value of 5 ft deduced from the test result.

Carl Rohwer,[12] M. ASCE.—The serious depletion of ground-water supplies in many areas during World War II has focused attention on the problems of ground-water hydrology. In this connection the investigations of the engineers of the Water Resources Branch of the U. S. Geological Survey are adding important information regarding the characteristics of wells and the capacity of ground-water formations. The analysis of drawdown tests of artesian wells by the author is a valuable contribution to this subject.

The writer is in agreement with the objectives of the author's investigations but he is of the opinion that the analysis of the problem would have been simplified if some of the factors that have only a slight effect on the results had been ignored. Under most conditions met with in the field of engineering the compressibility of water can be ignored. The coefficient is approximately 4×10^{-6} per pound pressure at ordinary temperatures and pressures. A reduction in pressure of 10 lb per sq in. would increase the volume of 1 cu ft of water by only 0.00004 cu ft, a difference of 1 in 25,000. In view of the large unavoidable errors involved in other measurements it seems that this factor could well be neglected. The same may be stated of the compression of the aquifer. As indicated by the author (see heading, "Application of Theory to a Simple Drawdown Test"), the combined effect of compressibility of the water-bearing formation is only five times the actual compressibility of water. Consequently, the combination of these two factors would produce a change of only 1 in 5,000 for a drop in pressure of 10 lb (approximately 23 ft). If the change in pressure were increased to 100 ft the effect produced by the compressibility of the water and aquifer would not be significant.

In reference to the tests on a shallow well at Meadville, Pa., the author states in the sentences following Eq. 18, that:

> "The foregoing calculations indicate that $B\,Q$ in this case was about 19.5 ft after 24 hours of continuous pumping. The observed drawdown in the pumping well at that time was 48.0 ft, leaving 28.5 ft for the well loss."

Such a large well loss seems unusual for an inflow of 1,350 gal per min through 15 ft of 18-in. screen unless the screen were badly encrusted or improperly perforated. Immediate steps should be taken to improve the performance of the screen in this well.

In the solution of problems involving many variables of which only a few can be determined by direct measurement, the use of multiple equations provides a method of determining the unknowns. However, there are difficulties inherent in this method which may lead to contradictory or inconsistent results.

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As shown by the author in reference to the determination of C (see heading, "Data from Multiple-Step Drawdown Test"), there is considerable scattering in the values obtained for the "specific incremental drawdown," Fig. 7(*d*). No doubt, this is due in part to the inaccuracies in the drawdown readings. If this method were used on problems in which all readings could be made accurately, consistent results should be expected. Since this is not generally true, the multiple-equation method results in solutions in which the final answers may have errors greatly in excess of the observed data. The writer is not aware of the mathematical basis for this assumption, but he observed the same effect when attempting to use a similar method to determine the values of the factors involved in the seepage from canals. The conclusion was reached that, in the elimination of variables from the series of equations by subtraction, the variables eliminated were forced to conform exactly to the law; and, as a result, all the discrepancies accumulated and finally appeared in the solution of the unknown. A solution based on another pair of equations may, therefore, yield a result widely different from the first one.

Since the author has had the opportunity to observe how the solutions vary when he uses different equations it would be of interest to study the mathematical principles causing the variations. No doubt rules could be formulated which would make it possible to obtain more consistent results from the observed data. Such an analysis would be useful in the solution of problems in other fields of engineering.

R. M. Leggette,[13] Affiliate, ASCE—Although it covers a highly technical subject, this paper clearly demonstrates the practical importance of a number of factors of well design. It seems desirable to emphasize these practical considerations because they are often given too little attention. Frequently water works men and well-drilling contractors greatly belittle or fail to recognize the magnitude of what Mr. Jacob calls "well loss."

It is obvious, of course, that the water level in a pumping well must be lower than the water level immediately outside the well. In many wells, much of this difference in head is screen friction loss which results from the use of a poorly designed screen. This difference in head is sometimes presumed to be only a few inches, or a fraction of a foot; however, actual observations have shown that in some wells the well loss is a considerable part of the total drawdown. Thus, from the point of view of economy of operation, well loss may be an important factor.

The paper also indicates the desirability of increasing the effective radius of a "sand and gravel" well by development to remove the fine material surrounding the screen, or by artificial gravel packing. It should be noted that the advantages of development or gravel packing may be largely overcome if an inefficient well screen is used.

The process of development by surging, swabbing, and brushing is being used more and more in uncased wells (rock wells), the walls of which apparently become "mudded up" during the drilling process. This clogging of the uncased wall of the well has the same effect as an inefficient well screen in a "sand

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and gravel" well. The well loss in many wells of this type has been greatly reduced by development.

From the point of view of economy of pumping well water, this paper indicates the following: A well should be of large diameter; it should be extensively developed; and, if a well screen is used, it should be designed so as to produce a minimum of screen friction loss.

M. R. LEWIS,[14] M. ASCE.—A valuable method has been presented, in this paper, for analyzing the capacity of artesian wells in spite of the necessary assumptions that there is uniformity in the aquifer and that the total supply to the well is drawn from the aquifer by the release of elastic forces. Such theoretical or mathematical treatments of the flow of fluids assist greatly in the understanding of practical problems even though the latter seldom are based on ideal conditions.

The multiple-step test proposed by the author should give very useful information on the points mentioned in the "Summary" wherever the assumptions are approximately fulfilled. It is hoped that the author will explore the possibility of a similar analysis under other conditions. Two important types of wells that might be studied are those of a simple water-table situation and those in which the bed overlying an aquifer is relatively impermeable but permits recharge from the soil surface surrounding the well.

It appears to the writer that two other factors besides the compaction of the aquifer are important elements in making the "apparent compressibility, β'," greater than the compressibility of water, β. These are (*a*) the increase in volume of the solid material of the aquifer because of the reduced hydrostatic pressure and (*b*) the reduction in the pore space because of the deformation of the solid particles by reason of the increased pressure on the mineral skeleton. In his earlier paper,[8] the author mentioned these factors but, apparently, considered them to be of negligible importance. Whether they are, or are not, important makes no difference in the author's analysis.

C. E. JACOB,[15] Assoc. M. ASCE.—Although few in number and brief, the discussions have added much to the paper and, moreover, have suggested the direction that further work might profitably take. The writer wishes to thank those who have contributed.

The question raised by Mr. Boulton is a pertinent one—regarding the need for evidence to support the assumption that the compressibility of the aquifer has the same value near to, and far from, the pumped well despite the wide range of pressure release. The writer knows of no close observations of long-period decline under constant and continuous pumping that might clarify this problem. Of course, it is to be expected that an unconsolidated sand would have a variable compressibility, depending on the rate and magnitude of the loading occasioned by the release of pressure. Moreover, sight should not be lost of the fact that the flexural rigidity of the overlying beds complicates the problem, especially in the immediate vicinity of pumped wells tapping deep aquifers. Fortunately, however, as long as the compressibility (thus modified)

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may be assumed reasonably constant in time at a given distance the uncertainties arising from ignorance of its absolute value are reflected only in the degree of approximation of r_w. Actually, in predictions of future drawdown based on the theory of an elastic aquifer, the product $S\,r^2_w$ is used. This product is determined empirically and it is not necessary to break it down into S and r_w, except to compare different wells. If S varies with t, for one reason or another, then some other theory must be used or the elastic theory must be modified.

Mr. Boulton's estimate of 10.5 ft for the friction and velocity head losses accompanying the upward flow inside the well casing of the Bethpage well narrows down the well loss to that part that may be termed "screen loss." His calculation of the "convergence loss" under laminar flow into an assumed system of vertical slots shows that such loss would be quite small. Actually the openings in the screen are in the form of a helix, but the "convergence loss" there should be of the same or even of a smaller order of magnitude. Clogging of the slots is a distinct possibility, as pointed out by Mr. Boulton. Furthermore, departure from laminar flow may begin as the water passes through the screen openings, especially if they are clogged.

By calculation Mr. Rohwer shows that the combined relative compression of the aquifer near Meadville, Pa., and its contained water is only about 1 in 5,000 for a 23-ft drop in head. He states:

> "In view of the large unavoidable errors involved in other measurements it seems that this factor [compressibility of water] could well be neglected. The same may be stated of the compression of the aquifer."

Perhaps Mr. Rohwer has in mind estimating the ultimate or steady-state discharge of wells in a limited aquifer, in which case the volume of water derived from artesian storage might soon become small by comparison with the volume drawn through the aquifer from an outer boundary. However, until that steady state of flow is established, water is withdrawn from storage through its elastic expansion and through the concomitant compression or compaction of the aquifer. Indeed, in extensive and deep-lying aquifers such as the Dakota sandstone, the supply may be furnished entirely from storage for decades. Because of the tremendous volume of water in such an aquifer and because of the sizable lowering of head that may be produced, the total volume of water withdrawn from storage through wells may itself reach an astounding magnitude. In any event, however insignificant this factor may appear, the study of the transient behavior of an elastic aquifer, on which the paper is based, requires an appraisal of its magnitude.

With reference to the large well loss of 28.5 ft at 1,350 gal per min in the well near Meadville, Pa., the suggestion is offered by Mr. Rohwer that the screen may be encrusted or improperly perforated. As the well was newly constructed when tested, it would seem that sand packing of the gravel envelope and of the screen might be the explanation. The formation is a uniform fine sand—difficult to handle in drilling and developing a well.

Mr. Rohwer adds a valuable point in remarking on the unavoidable magnification of errors through elimination of variables by subtraction from the series of equations. He justifiably emphasizes the need for care in treating such data as those presented in the paper. Whereas from a theoretical standpoint, with ideal data, the procedure outlined in the paper is sound; in practice it needs modification. Higher precision of measurement may warrant the assumption that the drawdown obeys the law $s_w = B\,Q + C\,Q^n$, introducing a third unknown, the exponent n (<2), to be determined by trial-and-error computation, or by graphical procedure together with the coefficients B and C.

An important point is raised by Mr. Leggette—that the advantages of gravel packing or developing a well may be offset by poor design or improper choice of screen. The writer feels that amassing empirical values of C and of r_w, together with pertinent data on the details of design and construction of wells, may eventually make possible the accurate appraisal of these various factors. The selection of screen type, slot opening, and gravel size—and even the determination of whether or not a gravel envelope is required—may be lifted from the realm of guesswork to a rational plane through the future study of existing and newly constructed wells and through the measurement of their characteristics of performance, due consideration being given to the transient behavior of the aquifer.

In emphasizing the magnitude of well losses, condemnation of the well driller is not intended, for much of the friction loss in and near a well is unavoidable and will never be entirely eliminated. Nevertheless, it behooves the engineer and the well-drilling contractor alike to strive for as efficient design and construction as possible to meet the stringencies of economic demands. The points summarized by Mr. Leggette thus are objectives toward which progress should be made.

Mr. Lewis suggests that similar analyses be made for unconfined flow under simple water-table conditions, and for confined flow in which recharge from the soil surface occurs through a relatively impermeable confining bed. To the writer's knowledge a satisfactory analysis of nonsteady unconfined flow has not been given. Even in the case where the storage coefficient (S, ultimately approaching "specific yield") is constant, there are insuperable difficulties. Only by analogy to confined flow, and then in cases where the maximum drawdown is but a small fraction of the initial depth of flow, has an approximate solution been obtained. It may be stated, however, that even in the absence of well losses the specific capacity of a water-table well would vary with the discharge, the curve of drawdown versus discharge at constant time being a parabola under certain approximative assumptions. Further work should be done on this problem, both in the laboratory and in the field.

A solution has been given for the nonsteady radial flow toward a steadily discharging well in a leaky confined aquifer.[16] The leakage is assumed proportional to the drawdown. Whether this is exactly the condition Mr. Lewis has in mind is not known, but in the early phase of a transient state such a system acts like an ideal elastic aquifer without leakage. Accordingly, a short multiple-step drawdown test could be analyzed under those conditions on the basis of the elastic theory, although long-term predictions would consider the leakage.

[16] "Radial Flow in a Leaky Artesian Aquifer," by C. E. Jacob, *Transactions*, Am. Geophysical Union, Vol. 27, 1946, Pt. II, pp. 198–205.

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TRANSACTIONS

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RIGID-FRAME STRUCTURES SUBJECT TO NONUNIFORM THERMAL ACTION

By Carl C. H. Tommerup,[1] Assoc. M. ASCE

With Discussion by Messrs. I. Oesterblom, Frank R. Higley, A. L. Miller, Charles O. Boynton, William A. Conwell, and Carl C. H. Tommerup.

Synopsis

The purpose of this paper is to clarify the problem of determining stresses and strains at any point in rigid frames whose members are subject to a nonuniform temperature differential; that is, structural members in which the temperature on one face is different from that on the other face. Many engineers are unaware of the exceedingly high stresses created by this quite common loading condition. The paper develops a rational setup which is of general scope since it is applicable to any type of framework.

This paper is divided into five parts, of which Sections 2, 3, and 4 contain the complete thermal analyses of three basic types of frames.

1. Introduction

Thermal stress analysis is of considerable importance in many branches of the metal industry in connection with castings, structural frames, etc., as well as in the field of such reinforced-concrete structures as flues, flue portals in plinths of stack foundations, hot liquid flumes, foundations for boilers, oil refinery furnaces, and blast furnaces.

Although it is generally recognized that reinforced concrete is far from being an ideal material in structures that are subject to high nonuniform temperature changes, it is continually used under these circumstances with reasonable success. The main reason for this is undoubtedly the fact that no other material is so readily applied to a variety of structures. It is equally well known, however, that a reinforced-concrete structure built in the shape of a rigid frame (a closed ring, for example), and then heated to a fairly high degree from within, is certain to develop a number of noticeable fractures along all outside edges. These fractures, which are caused entirely by thermal action,

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